

## Cosmic internal symmetry in a finite universe

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### Abstract

Cosmic internal symmetry (COINS) relates vacuum (V), dark (D) matter, baryons (B) and radiation (R). It implies conservation of the Friedmann integral  $A$  which is numerically the same for each of the energies:  $A \sim A_V \sim A_D \sim A_B \sim A_R$ . Evidence for COINS comes from the concordance data, including the WMAP data which also indicate that the co-moving space has a finite size. COINS 1) links the value of the vacuum density,  $\rho_V$ , to the total dark mass,  $M_D$ , the total baryonic mass  $M_B$ , and the total number,  $N_R$ , of ultrarelativistic particles:  $\rho_V/M_{Pl}^4 \sim (M_{Pl}/M_D)^2 \sim (M_{Pl}/M_B)^2 \sim N_R^{-4/3}$ ; 2) is behind the coincidence of the observed cosmic densities at the present epoch when the size of the universe  $a(t_0) \sim A$ ; 3) rules out significant deviations from spatial flatness; 4) might fix the initial amplitude,  $\Delta$ , of cosmic perturbations; 5) controls cosmic entropy  $D$  (per dark matter particle); 6) suggests a solution to the naturalness problem. The COINS nature is due to the electroweak-scale ( $M_{EW} \sim 1$  TeV) physics. The hierarchy number,  $X = M_{Pl}/M_{EW} \sim 10^{15}$ , provides the quantities involved with an appropriate universal measure:  $A \sim X^4 M_{Pl}^{-1}$ ,  $\rho_V \sim X^8 M_{Pl}^4$ ,  $M_D \sim M_B \sim X^4 M_{Pl}$ ,  $N_R \sim X^6$ ,  $\Delta \sim X^{-2}$  (at  $M_{EW}$  temperatures),  $D \sim X$ . If two extra dimensions of the submillimeter size  $R_*$  exist and eliminate the hierarchy effect, the vacuum density  $\rho_V \sim R_*^{-4}$ .

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## 1 Introduction

A recent analysis of the first-year data of the Wilkinson Microwave Anisotropy Probe (WMAP) (Bennett et al. 2003, Spergel et al. 2003) has suggested that the observed co-moving space of the Universe is finite, containing a finite amount of matter (Luminet et al. 2003). This is an expanding space, and its present-day size is comparable to the current radius of the visual horizon. The conclusion follows from the shape of the power spectrum of the cosmic microwave background (CMB) anisotropy at the lowest harmonics, or largest spatial scales. It is found that the quadrupole is only about one-seventh as strong as would be expected in an infinite flat space. A similar effect (while not so dramatic) is observed also for the octopole. The lack of power on the largest scales indicates most probably that the space is not big enough to support them. A special model – the Poincaré dodecahedral space of positive spatial curvature with the density parameter  $\Omega \simeq 1.013 > 1$  – is demonstrated to reproduce the observed shape of the spectrum better than do models of an infinite space (Luminet et al. 2003). The figure for the density parameter is compatible with the WMAP limitations:  $\Omega = 1.02 \pm 0.02$  (Bennett et al. 2003).

The result can be verified by further analysis of the WMAP data and the upcoming Planck data. It may be expected that – independently of the specific model with its rigid geometry and exact parameter  $\Omega$  – the major conclusion about a finite size of the real Universe will survive future tests.

Under this last assumption, I discuss below physics of a finite universe with a focus on a special correspondence between cosmic vacuum and matter in the finite-size space. This vacuum-matter relation is a kind of symmetry. As is well-known, symmetries are usually classified into geometrical and internal symmetries, in fundamental physics. Geometrical symmetries concern relations in space and time, while internal symmetries relate, for instance, different particles of like kind. Vacuum-matter symmetry is internal symmetry that relates vacuum and three basic forms of matter - dark matter, baryons,

radiation. Like other symmetries, this *cosmic internal symmetry* implies conservation of a certain quantity which is the *Friedmann integral*. Friedmann integral is a time-independent genuine characteristic of each type of cosmic energy. Its numerical value can be found with the use of observational data. This value proves to be the same, approximately, for vacuum, dark matter, baryons and radiation. The time-independent identity of the four values of the Friedmann integral is a phenomenological manifestation of cosmic internal symmetry.

In Sec.2, the Friedmann integral is introduced, and its numerical estimation is made on the basis of the recent concordance data, including the WMAP data on the CMB anisotropy power spectrum treated a lá Luminet et al. (2003); in Sec.3, a link to fundamental theory is discussed with the use of the standard physics of the early Universe; in Sec.4, cosmic internal symmetry is used for the study of the cosmic coincidence, the observed flatness of space, the large-scale perturbation amplitude, the cosmic entropy; in Sec.5, this symmetry is considered in the context of extra dimensions; summary is given in Sec.5.

## 2 The Friedmann integral (FINT)

The dynamical equation of the Friedmann cosmology is based on differential geometry, and so it is valid independently of any topology. It may be written for the radial size,  $a(t)$ , of a finite co-moving space in the form:

$$\dot{a}^2 = (A_V/a)^{-2} + A_D/a + A_B/a + (A_R/a)^2 - k. \quad (1)$$

In a finite-size universe with positive spatial curvature (as in the model by Luminet et al. 2003), the size  $a(t)$  may be equal to the spatial curvature radius; in this case  $k = 1$ , in the Friedmann equation. Generally, the size and the radius of curvature may be different, and in such a case,  $k = (M/M_C)^{2/3}$ , where  $M$  is the total nonrelativistic matter mass (dark matter and baryons) in the universe and  $M_C$  is the matter mass within the volume with the radial size equal to the curvature radius. For a finite-size universe of negative spatial curvature,  $k = -(M_B/M_C)^{2/3}$ , in the equation above. For a finite-size flat space,  $k = 0$ . (Hereafter the speed of light  $c = 1$ .)

The constants  $A$  in Eq.(1) represent the four major cosmic energies which are vacuum (V), dark matter (D), baryons (B) and radiation, or relativistic (R) energy. The constants are the integrals of the Friedmann ‘thermodynamical’ equation:

$$A = [\kappa \rho a^{3(1+w)}]^{-\frac{1}{1+3w}}, \quad (2)$$

where  $\kappa = \frac{8\pi G}{3}$  and  $G$  is the gravitational constant;  $w$  is the pressure-to-density ratio which is  $-1, 0, 0, 1/3$  for vacuum, dark matter, baryons and radiation, respectively. The integral  $A$  is the *Friedmann integral* (hereafter FINT).

It is obvious that the FINT defined by Eq.2 for dark matter or baryons is directly associated with the total masses of dark matter,  $M_D$ , or baryons,  $M_B$ , in the finite-size universe:

$$A_D = 2\kappa M_D, \quad A_B = 2\kappa M_B. \quad (3)$$

Similarly, the FINT defined for radiation is determined by the total number of CMB photons,  $N_{CMB}$ , in the finite-size universe:

$$A_R \propto N_{CMB}^{2/3}. \quad (4)$$

As for the FINT for vacuum, it is given by the vacuum density:

$$A_V = (\kappa \rho_V)^{-1/2}. \quad (5)$$

Due to its origin from the Friedmann thermodynamical equation as a constant of integration, the FINT is not restricted by any theory constraints (except for trivial ones), and its values for various energy forms are completely independent of each other *a priori*.

The four values of the FINT can be evaluated numerically with the use of the current concordance figures for the four energy densities (Riess et al.1998, Perlmutter et al. 1999, Bennett 2003, Spergel 2003):

$$\Omega_V = 0.7 \pm 0.1, \quad (6)$$

$$\Omega_D = 0.3 \pm 0.1, \quad (7)$$

$$\Omega_B = 0.02 h_{100}^{-2}, \quad (8)$$

$$\Omega_R = 0.6 h_{100}^{-2} \alpha \times 10^{-4}, \quad 1 < \alpha < 10 - 30. \quad (9)$$

The figures are in general agreement with the Hubble parameter  $h_{100} = 0.70 \pm 0.15$  and the cosmic age  $t_0 = 14 \pm 1$  Gyr.

With these data, the FINT for vacuum

$$A_V = (\kappa\rho_V)^{-1/2} = \Omega_V^{-1/2} H^{-1} \simeq 3 \times 10^{28} \text{ cm}. \quad (10)$$

The constant quantity is near the Hubble radius,  $H(t_0)^{-1}$ , at the present epoch.

The three other FINT values are estimated with the use of the present-day size  $a(t_0)$  of the finite universe. The WMAP data on the power spectrum at lowest harmonics suggest that the size must be near the Hubble radius:

$$a(t_0) \sim H^{-1} \simeq 10^{28} \text{ cm}. \quad (11)$$

The double approximate identity  $a(t_0) \simeq H^{-1} \simeq A_V$  is a specific physical characteristic of the current epoch in the cosmic evolution.

With the size  $a(t_0) \simeq H^{-1}$ , one has

$$A_D = \kappa\rho_D a^3 = \Omega_D a^3 H^2 \simeq \Omega_D H^{-1} \simeq 3 \times 10^{28} \text{ cm}. \quad (12)$$

$$A_B = \kappa\rho_B = \Omega_B a^3 H^2 \simeq \Omega_B H^{-1} \simeq 3 \times 10^{27} \text{ cm}. \quad (13)$$

$$A_R = (\kappa\rho_R)^{-1/2} a^2 = (\Omega_R \alpha)^{-1/2} a^2 H \simeq 3 \times 10^{26} \text{ cm}, \quad (\alpha \simeq 10). \quad (14)$$

Note that, instead of the value  $A_R$ , three values may be introduced to the Friedman equation to account separately for the CMB photons,  $A_{CMB}$ , neutrinos  $A_\nu$ , other relativistic relics,  $A_{RR}$ , of the early cosmic evolution. It is obvious that  $A_{CMB} \sim A_\nu \sim A_R$ . As for  $A_{RR}$ , no direct data is yet available; one may only expect that  $A_{RR} \sim A_R$ , if, for instance, relic cosmological gravitons really exist.

### 3 Cosmic internal symmetry (COINS)

As we see, all the four FINT values calculated for the finite universe with the concordance data prove to be identical, on the order of magnitude:

$$A_V \sim A_D \sim A_B \sim A_R \sim A \sim 10^{60 \pm 1} M_{Pl}^{-1}. \quad (15)$$

Here units are used in which the speed of light, the Boltzmann constant and the Planck constant are all equal to unity:  $c = k = \hbar = 1$ . The Planck mass is  $M_{Pl} = G^{-1/2} \simeq 1.2 \times 10^{19}$  GeV.

The time-independent identity of Eq.15 may be treated as a symmetry relation that brings vacuum into correspondence with matter (as it was earlier proposed, – Chernin 2001, 2002a). Vacuum-matter symmetry does not relate to space and time, and therefore it is a type of internal symmetry. Symmetry between the proton and the neutron in nuclear physics is a basic example of internal symmetry: the particles are different in mass, electric charge, etc., but characterized by the same charge in nuclear (strong) interaction. The *cosmic internal symmetry* (hereafter COINS) is the symmetry of the cosmic energies: they are different in physical state, content, temporal behavior, etc., but each of them is characterized by the same (approximately) quantity which is the FINT.

COINS comes from the phenomenological analysis of the available observational data, and it reveals most naturally, as is seen from Sec.2, in the finite-size universe.

COINS is not an exact symmetry; it is valid on the order of magnitude and violated at a level of a few percent, on the logarithmic scale:  $\lg(A_V/A_R)/\lg A_V \simeq 0.03$ , with  $A$  in the Planck units. (According to Okun (1985), the notion of symmetry is essentially close to the notion of beauty. Furthermore, true beauty in its supreme forms requires a slight deviation from symmetry which communicates to beauty a mysterious and alluring element of non-finito.) COINS itself and its violation are both important for cosmological problems discussed below (Sec.4).

COINS is a covariant symmetry: it is formulated in terms of the FINT which is a scalar (invariant) quantity. FINT is associated with the four-dimensional Riemann invariant,  $R = 8\pi G(\rho - 3p)$ . In the limit of infinite time  $R \rightarrow 32\pi G\rho_V = 12/A_V^2$ ,  $t \rightarrow \infty$ . If cosmic matter is initially generated in the form of massless particles (and the particles acquire mass later via, say, the Higgs mechanism), the invariant is the same in the opposite time limit as well:  $R \rightarrow 12/A_V^2$ ,  $t \rightarrow 0$ .

COINS is a time-independent symmetry in the evolving Universe; it exists unchanged during all the time when the observed cosmic energies exist themselves. Like most of

symmetries, COINS corresponds to the conservation of the appropriate quantity: this is obviously the FINT which is a genuine ‘charge’ of this symmetry.

COINS is a robust symmetry: it is not sensitive to (not too big) variations of the figures involved. For instance, the FINT for the non-vacuum energies may be re-estimated with the use of another, than in Sec.2, value of  $a(t_0)$ . It may be equal, say, to the curvature radius in the model with the present-day  $\Omega = 1.013$ , as suggested by Luminet et al.(2003):

$$a(t_0) = a_C(t_0) = (\Omega - 1)^{-1/2} H^{-1} \simeq 10 H^{-1} \simeq 10^{29} \text{ cm}. \quad (16)$$

Then one will have, instead of Eq.15:

$$A_V \sim A_D \sim A_B \sim A_R \sim A \sim 10^{62 \pm 1} M_{Pl}^{-1}. \quad (17)$$

An approximate identity of the four values of the FINT takes place for the new figures as well.

The physical nature of COINS can be clarified and the symmetry relation for vacuum, dark matter and radiation can be explained by (or even deduced from) a freeze-out model (Chernin 2001, 2002a). The model is based on the standard physics of the early universe and assumes that the weakly interacted dark matter particles (WIMPs) have a mass near the the fundamental electroweak energy scale  $m_D \sim M_{EW} \sim 1 \text{ TeV}$ . Freeze-out of dark matter annihilation occurs at temperatures  $\sim m_D \sim 1 \text{ TeV}$  when the red shift  $z \sim M_{Pl}/M_{EW}$ . The model provides COINS with a link to fundamental physics: it indicates that the FINT can be represented in terms of the two fundamental energies, which are  $M_{Pl}$  and  $M_{EW}$ , only:

$$A \sim (\bar{M}_{Pl}/M_{EW})^4 M_{Pl}^{-1} \simeq 10^{61 \pm 1} M_{Pl}^{-1}. \quad (18)$$

Here the reduced Planck scale  $\bar{M}_{Pl} = g M_{Pl}$ ,  $g \simeq 0.1 - 0.3$ , is introduced to account (as usual) for some dimensionless factors (like  $8\pi/3$ , etc.) of the theory.

According to the model, the constant vacuum density,

$$\rho_V \sim (\bar{M}_{Pl}/M_{EW})^{-8} M_{Pl}^4 \simeq 10^{-122 \pm 2} M_{Pl}^4, \quad (19)$$

is expressed in terms of the same fundamental energies (see also Arkani-Hamed et al. 2000).

The ratio  $\bar{M}_{Pl}/M_{EW} \sim 10^{15}$  that enters the result is known as the *hierarchy number* in particle physics. It characterizes the huge gap between the two fundamental energies; the nature of the gap is not well understood, and this is considered as one of the central problems in fundamental theory. We address this problem in Sec.4 below.

Note that the FINT for baryons is not treated by the freeze-out model. It may, however, be assumed that the physics of baryogenesis is basically behind the relation between the FINT for baryons and the FINT for dark matter (Chernin 2001). Baryogenesis and associated physics might also be responsible (at least, in part) for the COINS violation.

## 4 Solving problems with COINS

Let us turn now to some old and new problems in cosmology and show how cosmic internal symmetry may help in their better understanding.

### 4.1 Cosmic coincidence

Why are the observed densities of vacuum, dark matter, baryons and radiation are nearly coincident? This is the *cosmic coincidence problem* which has appeared with the discovery of cosmic vacuum (Riess et al.1998, Perlmutter et al. 1999) and is usually considered as a severe challenge to current cosmological concepts (see, for instance, Chernin 2002a and references therein). As is well-known, the idea of quintessence was introduced in an attempt to eliminate the problem. However it is now clear that quintessence can hardly be useful because the pressure-to-density ratio has recently been found to lie between -1.2 and -0.9 (Perlmutter et al. 2003), which seemingly rules out the idea.

On the contrary, the approach based on COINS offers a natural solution to the problem without any additional assumptions. Indeed, one may easily see that the four densities can become identical (approximately), because the four values of the FINT are



identical (approximately):

$$\kappa\rho_V \sim A^{-2}, \kappa\rho_D \sim (A/a)^3, \kappa\rho_B \sim (A/a)^3, \kappa\rho_R \sim (A/a)^4. \quad (20)$$

The densities happen to coincide now-days, because  $a(t_0) \sim A$  right now.

Thus, the cosmic densities are observed to be nearly coincident, because of COINS and the special character of the moment of observation at which  $a(t_0) \sim A$ .

## 4.2 The Dicke problem

The geometry of the co-moving space looks nearly flat in observations. Why this is so? The *problem of flatness* was recognized by Dicke (1970) and formulated in terms of the time-dependent parameter  $\Omega(t)$ . As Dicke (1970) mentioned, the universe must be extremely finely tuned to yield the observed balance between the total energy density of the Universe and the critical density. This balance must be tuned with the accuracy  $\sim 10^{-16}$  or  $\sim 10^{-60}$ , if it is fixed at the epoch of the light element production or at the Planck epoch, respectively.

COINS shows the problem in quite different light. With the use of simple relations based on the Friedmann equation of Sec.2 (see also Chernin 2003) one may find that the maximal possible deviation from  $\Omega = 1$  in a universe with the space of positive or negative curvature occurs at  $z = a(t_0)/(\frac{1}{2}A_V^2 A_D)^{1/3} - 1 \simeq 1$ . The deviation goes to zero in both limits  $t \rightarrow 0$  and  $t \rightarrow \infty$ , as is seen directly from the Friedmann equation.

The maximal possible deviation is expressed in term of the FINT values for vacuum and dark matter:

$$|\Omega_{ex} - 1| \simeq |[1 - \frac{1}{2}(\frac{A_V}{A_D})^{2/3}]^{-1} - 1|, \quad (21)$$

in the case when the finite space has positive curvature and its size is the curvature radius (see Sec.2). The most severe WMAP limits,  $\Omega - 1 = 0.02 \pm 0.02$ , are met, if  $A_V/A_D \leq 0.008$ . This is possible when  $a(t_0) \geq (5 - 10)H^{-1}$ . The model proposed by Luminet et al. (2003) is a concrete example which fits this condition.

In a more general case, one has, in accordance with Sec.2,

$$|\Omega_{ex} - 1| \simeq |[1 \pm \frac{1}{2}(\frac{A_V}{A_D})^{2/3}(M/M_C)^{2/3}]^{-1} - 1|, \quad (22)$$

It is seen from here that a finite space of positive curvature fits the WMAP data, if, say,  $A_V/A_D \simeq 1$  and the present-day curvature radius  $a_C(t_0)$  is 5 times the radial size of the space  $a(t_0)$ .

To feel the contrast with the fine-tuning argument, one may compare the modest numbers between 3 and 10 with the enormous numbers  $10^{-16}$  or  $10^{-60}$ .

Thus, the interplay between vacuum antigravity and dark matter gravity at  $z \sim 1$  is behind the observed near flatness of the space. The interplay is controlled by COINS, and COINS rules out completely any significant deviations from flatness at present, in the past and future of the Universe. The exact quantitative measure of a possible non-flatness is determined by both COINS itself and its violation, – without any fine tuning.

Note that no hypothesis (about, say, an enormous vacuum density, or enormous energy density of inflaton field, at very large  $z$ ) is required to understand the physics of the Dicke problem. The really observed vacuum density and simple cosmology at modest  $z$  are quite enough to understand why the observed space looks nearly flat.

### 4.3 Perturbation amplitude

Cosmic perturbations that give rise to structure formation are characterized by their spectrum and initial amplitude. The spectrum can be obtained in the theory of quantum fluctuations developed first by Chibisov and Mukhanov (1981); they obtain the spectrum which is similar in shape to the Harrison-Zeldovich spectrum and well confirmed by the WMAP and other recent data. However, there is no theory yet that could give the perturbation initial amplitude.

In the theory of the perturbation evolution, one faces another fine-tuning problem which nevertheless seems to be fairly similar to that in the Dicke argument. Indeed, the perturbations must be extremely finely tuned in amplitude to come to the nonlinear regime between the red shifts, say,  $z \simeq 3 - 10$  and  $z \simeq 1$  (at  $z < 1$  vacuum antigravity terminates the perturbation growth).

Consider, for instance, an example (Chernin 2002b) of large-scale adiabatic perturbations which are ever over-horizon in size and ever grow before  $z \sim 1$ . Such scales are

larger than 10-30 Mpc at the present epoch (and may also be near the largest scales related to the low-harmonic end of the CMB anisotropy power spectrum – see Sec.1). If the perturbations are generated at the epoch of light element production or at the Planck epoch, their initial amplitudes (estimated with the standard theory) must be tuned with the accuracy  $10^{-16}$  or  $10^{-60}$ , respectively.

COINS suggests a new approach to the problem. To start with, one may consider the large-scale perturbation as above and use, in addition, an idea by Zeldovich (1965). In his study of weak adiabatic perturbations, Zeldovich found that the correct time rate of the perturbation growth could be obtained in a simple picture in which a perturbation was treated as an area of negative total energy on the unperturbed background of the parabolic expansion, – if to use the language of the Newtonian mechanics. In other words, any such perturbation may be considered as a part of a universe with a co-moving space of positive curvature. Assume also that the FINT values are the same in the perturbation area and in the background model. Then, extending the Zeldovich idea in this way, one may find that the perturbations come indeed to the nonlinear regime just at about  $z \sim 1$ .

To see this, one may address again the relations of the subsection above. Indeed, the relative amplitude of density perturbations is obviously related to the density parameter  $\Omega$ , in the Zeldovich picture:

$$\delta\rho/\rho \simeq \Omega - 1, \quad (23)$$

It reaches its maximum value at  $z \sim 1$ , and at that time

$$\delta\rho_D/\rho_D = \Omega_{ex} - 1 \simeq [1 - \frac{1}{2}(\frac{A_V}{A_D})^{2/3}(M/M_C)^{2/3}]^{-1} - 1. \quad (24)$$

The value is about unity,  $\delta\rho_D/\rho_D \sim 1$ , provided  $(A_V/A_D)(M/M_C) \sim 1$ . The latter requirement may, for instance, has the form:  $A_V/A_D \sim 1$ ,  $M/M_C \sim 1$ .

As we see, the perturbations become nonlinear at  $z \sim 1$  simply because of COINS. No fine tuning in amplitude is needed, and no (explicit) assumption about the perturbation amplitude is involved in the considerations. The basic assumption is only the validity of COINS in both perturbation and background. If perturbations are generated in this special way, their correct time behavior is guaranteed: they reach the nonlinear regime

on time.

(It must be, however, assumed also that the perturbation area and the background start to expand at more or less the same moment of time. It would be enough, actually, if the difference in the start time,  $\delta t$ , is, say, several times less than the cosmic age at the epoch of  $z \sim 1$ . The effect of asynchronous start diminishes with time as  $\delta t/t$ ; this is the so-called falling perturbation mode, which is not too significant in any reasonable circumstances.)

The perturbation evolution and its time-dependent amplitude are given by Eq.23; one may use it to estimate the amplitude of this type perturbations at, say, the freeze-out temperature  $\sim M_{EW}$  (see Sec.3) when the red shift  $z = z_{EW} \sim \bar{M}_{Pl}/M_{EW}$ :

$$\Delta(z_{EW}) \simeq \Omega(z_{EW}) - 1 \simeq [a(t_0)/A_R]^2 (1+z)^{-2} \sim (M_{EW}/\bar{M}_{Pl})^2 \sim 10^{-30}. \quad (25)$$

Thus, COINS shows how initial perturbations might be prepared without any fine tuning. The freeze-out model associated with COINS gives the perturbations a natural initial amplitude in terms of the two fundamental energies: this is the inverse squared the hierarchy number, at the electroweak temperatures.

Remind that the considerations concern the perturbations of ever over-horizon spatial scales. But in their physical nature, the perturbations on these scales are hardly different essentially from the perturbations on all the other scales. The theory by Chibisov and Mukhanov (1981) gives rather an evidence for a common nature of the perturbations over their whole scale range. If so, physics which is responsible for the origin of the cosmic perturbations involves COINS and the hierarchy effect as its basic ingredients.

#### 4.4 Cosmic entropy

The number density of the CMB photons  $n_R \sim 1000$ , at present. The baryon number density  $n_B \sim 10^{-6}$  now. The time-independent ratio,  $B = n_R/n_B \sim 10^9$ , is called the Big Baryonic Number; as is well-known, it represents the cosmic entropy per baryon. Why this number is so big? This question is referred to as the *cosmic entropy problem*.

In terms of the FINT, the Big Baryonic Number may be represented as

$$B \sim A_R^{3/2} A_D^{-1} m M_{Pl}^{-1/2}. \quad (26)$$

If to use COINS as an exact symmetry and put  $A_R = A_B$ , one has from here  $B \sim 10^{11}$ , which is not too bad as the first approximation. If not only symmetry, but its violation as well is taken into account, the number will obviously be quite correct.

The freeze-out model suggests that the *Big Dark Number* may also be of interest:

$$D = n_R/n_D \sim 10^{12}, \quad (27)$$

where  $n_D$  is the number density of dark matter particles and it is assumed again that the WIMP mass  $m_D \sim M_{EW}$ . In terms of the FINT, one has

$$D \sim A_R^{3/2} A_D^{-1} M_{EW} M_{Pl}^{-1/2}. \quad (28)$$

For exact symmetry relation,  $A_R = A_D$ , and with the FINT expressed via the two fundamental energies, this gives

$$D \sim \bar{M}_{Pl}/M_{EW} \sim 10^{15}, \quad (29)$$

which is the simplest (and perhaps ‘more fundamental’ than  $B$ ) measure for the cosmic entropy per particle.

Thus, in the first approximation, one may answer the question above: the cosmic entropy per particle is given by a big number because of COINS and the hierarchy effect in fundamental physics. COINS and the hierarchy control – via cosmic entropy – cosmic light element production in the Big Bang.

## 5 COINS, hierarchy and extra dimensions

As we see, the hierarchy number provides COINS and the associated phenomena with a common quantitative measure: the basic constants of cosmology which are the FINT, cosmic entropy per particle, the initial perturbation amplitude are expressed via this big number. Implicitly this number is also involved in the phenomena of cosmic coincidence and flatness. Therefore, if the hierarchy problem is resolved in fundamental physics, it will give a new insight into the nature of COINS.

The hierarchy problem has been discussed in fundamental physics for decades. We will not consider here earlier ideas and address rather a recent one by Arkani-Hamed

et al. (1998). This is the idea of macroscopic extra dimensions which are proposed to eliminate the energy hierarchy of fundamental theory.

The idea assumes that there exist finite (compactified) submillimeter extra dimensions in space, and all the dimensions constitute a close multi-dimensional space. This multi-dimensional space is treated as the *true space of nature*. It is also assumed that there is one and only one *truly fundamental* energy scale  $M_*$  in nature, and it is close to the electroweak scale  $M_{EW}$ . As for the Planck scale, it is reduced to a combination of the scale  $M_*$  and the size  $R_*$  of the compact macroscopic extra dimensions of the true space:

$$M_{Pl} \sim (M_* R_*)^{n/2} M_*. \quad (30)$$

Here  $n$  is the number of the spatial extra dimensions, which are proposed to be of the same size. It is reasonably argued that the case  $n = 2$  is the most appropriate one; if so, the size of two extra dimensions is in the millimeter (or rather submillimeter) range:

$$R_* \sim 0.1 \text{ cm}, \quad n = 2. \quad (31)$$

Together with the Planck mass, the gravitational constant in three-dimensional space,  $G = M_{Pl}^{-2}$ , loses its fundamentalism and is reduced to the two truly fundamental constants  $M_*$  and  $R_*$ .

We will not go here into further details of the extra-dimension physics and adopt from it only one result: the hierarchy number,  $M_{Pl}/M_{EW}$ , is replaced with the product  $M_* R_*$ . This is actually not elimination of the hierarchy, but its re-formulation in the new terms of  $M_*$  and  $R_*$ .

In the multi-dimensional space, all the physical fields, except gravity, are assumed to be confined in the three-dimensional space, or brane. Therefore the Friedmann theory (together with the FINT, COINS,  $B$ ,  $D$ ,  $\Delta$ , etc.) is defined at the *cosmological brane* of finite sizes. The multi-dimensional physics affects the brane, and therefore cosmology should be re-formulated in terms of the true fundamental constants. In this way, one has for the FINT:

$$A \sim (M_* R_*)^{(3/2)n} M_*^{-1}. \quad (32)$$

In the case of two extra dimensions:

$$A \sim (M_* R_*)^2 R_*, \quad n = 2. \quad (33)$$

Then the vacuum density

$$\rho_V \sim (M_* R_*)^{-2n} M_*^4, \quad (34)$$

and in the case of two extra dimensions:

$$\rho_V \sim R_*^{-4}; \quad n = 2. \quad (35)$$

This is the most surprising result: the vacuum density proves to be expressed via the size of the extra dimensions alone. The new relation is free from any signs of the hierarchy effect (Chernin 2002c). This is a case when the hierarchy is really eliminated from the multi-dimensional physics.

According to the idea of extra dimensions, all we observe in three-dimensional space are shadows of the true multi-dimensional entities. In particular, it may be assumed that true vacuum exists in the multi-dimensional space, and the observed cosmic vacuum is not more than its 3D projection to the cosmological brane. This is possible, if vacuum is due to gravity only and not related to the fields of matter. In this case, the true vacuum is defined in the multi-dimensional space, and its density  $\rho_{V5} \sim R_*^{-6}$ , in the case of two extra dimensions. This true vacuum is also free from the hierarchy effect. Via vacuum, COINS brings the energies on the brane in a correspondence with the physics of the multi-dimensional world.

Turning to the topics of Sec.3 above, one finds for the Big Dark Number:

$$D \sim M_* R_*, \quad n = 2. \quad (36)$$

In a similar way, the initial (at the electroweak temperatures) amplitude of the large-scale perturbations may be found:

$$\Delta \sim (M_* R_*)^2, \quad n = 2. \quad (37)$$

Taking the relations at face, one may conclude that the basic cosmological parameters have roots in the extra-dimension physics, – if extra dimensions really exist. It is

expected that their existence will be directly tested with the TeV hadron collider and in submillimeter laboratory experiments in several years – perhaps, at the same time when the Planck mission will test compactness on cosmological scales.

## 6 Discussion and Conclusions

Cosmic internal symmetry (COINS) is a time-independent phenomenon in the evolving Universe. COINS unifies vacuum and matter, revealing their internal correspondence and perhaps common physical nature. Evidence for the existence of COINS comes in a natural way from the concordance observation data. The evidence is most straightforward, if the size of the co-moving space Universe is finite. The WMAP data in the interpretation by Luminet et al. (2003) do suggest that the Universe is finite, containing a finite matter mass.

COINS imply that the vacuum density  $\rho_V$  is in a direct quantitative inter-relation with the total mass of dark matter,  $M_D$ , the total mass of baryons,  $M_B$ , and the total number of ultrarelativistic particles (first of all, the CMB photons and neutrinos),  $N_R$ , in the finite Universe:

$$\rho_V M_{Pl}^{-4} \sim (M_{Pl}/M_D)^2 \sim (M_{Pl}/M_B)^2 \sim N_R^{-4/3}. \quad (38)$$

The freeze-out model based on the standard physics provides COINS with a link to fundamental theory. As a result, the dimensionless hierarchical number,  $X = \bar{M}_{Pl}/M_{EW} \sim 10^{15}$ , emerges and gives all the quantities involved a universal measure:

$$A \sim X^4 M_{Pl}^{-1}, \quad \rho_V \sim X^{-8} M_{Pl}^4, \quad M_D \sim X^4 M_{Pl}, \quad M_B \sim X^4 M_{Pl}, \quad N_R \sim X^6. \quad (39)$$

If the hierarchy effect of fundamental physics roots in (two) macroscopic extra dimensions (Arkani-Hamed et al. 1998), the true energy  $M_* \sim 1$  TeV and the submillimeter size of extra dimensions  $R_*$  come to the cosmology scene; then  $X \sim M_* R_*$ , and the relations take the form:

$$A \sim X^2 R_*, \quad \rho_V \sim R_*^{-4}, \quad M_D \sim X^5 M_*, \quad M_B \sim X^5 M_*, \quad N_R \sim X^6. \quad (40)$$



It has been long recognized (see Okun, 1985, and references therein) that the energy scale  $M_* \sim 1$  TeV plays a central part in fundamental physics. Now COINS shows that, in this way or another, the energy proves to be most significant as well in cosmology.

Other results of the discussion may be briefly summarized as follows:

- \* The present-day size of the expanding space is equal or close, on the order of magnitude, to the value of the FINT:  $a(t_0) \sim A$ . This is a special feature of the present epoch of the cosmic evolution.

- \* COINS is behind the coincidence of the four observed cosmic density, and the phenomenon takes place just now, because  $a(t_0) \sim A$  at present.

- \* COINS controls the interplay between matter gravity and vacuum antigravity in space-time; as a result, it rules out any considerable deviations from spatial flatness now, in the past, and in the future of the Universe.

- \* COINS might be involved in the generation mechanism for cosmic perturbations; it implies that the perturbation initial (at the epoch of  $\sim 1$  TeV temperatures) amplitude  $\Delta \sim X^{-2}$ , is given in terms of the hierarchy number.

- \* COINS is responsible for cosmic entropy, so that entropy per dark matter particle, the Big Dark Number,  $D = n_R/n_D \sim X$ , is also measured by the hierarchy number. Indirectly, via entropy, COINS controls the outcome of the cosmic light element production in the Big Bang.

- \* Finally, COINS suggests a solution to the long standing *naturalness problem*. It shows that the vacuum density is as it is, because all the values of the FINT, including the value for vacuum, are identical. In addition, COINS together with extra-dimension physics lead to a fairly natural (and the simplest) relation:  $\rho_V \sim R_*^{-4}$ .

In principle, almost all the results above may equally be formulated for any reasonable cosmological model compatible with the current concordance data in both finite or infinite space. Specific data may be used as well. For instance, the WMAP figure for  $\Omega = 1.02 \pm 0.02$  suggests a close space, and the corresponding present-day curvature radius ( $\simeq 6 \times 10^{28}$  cm) may easily replace the size  $a(t_0)$  in the relations of Secs.2-3. This will not, however, alter the symmetry relation or change further results in any significant way. Another version is the simplest flat-space model with infinite space; in this case,

the FINT evaluation can be made with the use the co-moving size of the Metagalaxy, the currently visible part of space, ( $\simeq 10^{28}(1+z)^{-1}$  cm ), – and again it will not change the results. In this sense, COINS is a robust and model-independent result of cosmic phenomenology.

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